

Roll No.

Total Pages : 3

BCA/M-15

19018

MATHEMATICAL FOUNDATIONS-II

Paper-BCA-206

Time Allowed : 3 Hours]

[Maximum Marks : 80

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. Question No. **9** is compulsory.

UNIT-I

1. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, find $\frac{dy}{dx}$. 8

(b) If $x^y + y^x = c$, find $\frac{dy}{dx}$. 8

2. (a) If $y^{1/m} + y^{\frac{1}{m}} = 2x$, prove that

$$(x^2 - 1) y_{n+2} + (2n + 1) x y_{n+1} + (n^2 - m^2) y_n = 0. \quad 8$$

(b) Show that the two circles $x^2 + y^2 + 2nx + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch, if $a^2 + b^2 = c^{-1}$. 8

UNIT-II

3. (a) Expand $\tan x$ in power of $\left(x - \frac{\pi}{4}\right)$. 8

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- (b) Find the Asymptotes of $(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0$. 8

4. (a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$. 8

- (b) Find the position and the nature of the double points are the curve $y^2 = (x - 2)^2(x - 1)$. 8

UNIT-III

5. (a) If C_x and C_y be the chords of curvature parallel to co-ordinates axis at any point of curve $y =$

$$C \left(\frac{e^{x/c} + e^{-x/c}}{2} \right); \text{ prove that } 4 C_x^2 (C_x^2 + C_y^2) = C_y^4. \quad 8$$

- (b) Trace the curve $x^2 = (y - 1)(y - 2)(y - 3)$. 8

6. (a) Find the radius of curvature for the curve

$$\sqrt{x} + \sqrt{y} = \sqrt{a} \text{ at the point } \left(\frac{a}{4}, \frac{a}{4} \right). \quad 8$$

- (b) Trace the curve $r = a(1 + \sin \theta)$. 8

UNIT-IV

7. (a) If $u = f(r)$ and $x = r \cos \theta$; $y = r \sin \theta$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. 8

- (b) If $u = x \phi(y/x) + \psi(y/x)$; show that

$$\frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x^2 \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0. \quad 8$$

8. Find the extreme values of the function $x^2 + y^2 + z^2$ subject to the condition $xy + yz + 3x = 3a^2$. 16

UNIT-V

Compulsory Question

9. (a) Define Radius of curvature of a curve. 2

- (b) If $y = x^2$, find $\frac{dy}{dx}$. 2

- (c) Find nth derivative of $\frac{1}{ax + bi} x \neq -b/a$. 2

- (d) At point on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$ the tangent is parallel to x-axis. 2

- (e) State Taylor's theorem with Lagrange's form of Remainder after n-terms. 2

- (f) Expand $\log x$ in terms of x . 2

- (g) Find the asymptotes of the curve $y = \frac{x}{x^2 - 1}$. 2

- (h) Define implicit function. 2